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Multiplex realism

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Abstract

According to the view of *multiplex realism*, which will be presented and motivated in this article, our three-dimensional Euclidean theater is only one among many theaters that can be conceived and constructed, to stage the whole of our reality. The view of a 'multiplex reality' has consequences not only for our understanding of the nature of the physical world, particularly when we consider the relation between classical and non-classical (quantum and relativistic) entities, but also for our understanding of the manifestations of consciousness.

1 Introduction

As an evolving species, we have been present for hundreds of thousands of years in a very special niche of our reality: the crust of our beautiful planet Earth, surrounded by very special physical entities. From our multiple interactions with these entities, which we experienced by means of our dense bodies, we started, a long time ago, the construction of a prototypical *worldview*, in the attempt to order our experiences into a consistent *map of relations*. From this pre-cultural and pre-scientific construction, a first *clothing and decoration* of our reality resulted, allowing us to identify those portions of it that were recognizable as *aggregates of sufficiently stable properties*, where by 'stable' we mean that these properties could remain *actual* for enough time, thus becoming easy to observe. These *clusters of relatively permanent properties* (think of a material entity having a given size, weight, etc.) are what are called today, in physics, *classical entities*, or *macroscopic objects*, which include the astronomical bodies that we can see moving in the sky, like the Moon and the Sun, obeying with good approximation the laws of non-relativistic classical mechanics.

We can distinguish (at least) two different *directions of penetration* in our process of clothing and decoration of reality. One direction, which we have just mentioned, is a *penetration in depth*, through which we have identified those phenomena that, according to our senses, particularly those of sight and touch (Aerts, 2014), stood out compared to others, because of their *availability* in interacting with our body and becoming part of our experiences, and because such availability persisted long enough, allowing us to have multiple experiences with them. In other terms, by means of our penetration in depth of reality, we have recognized the existence of *experientially separated and stable* portions of it, today called, as we said already, classical entities.

The second, in a sense complementary, direction of penetration, can be called *penetration in width*. It corresponds to our effort to organize the content of our experiences with all these different *aggregates of stable properties*, i.e., with the different physical entities that appeared to us to be separated, in the sense of not influencing each other in a significant way. This process of penetration in width, through which we have identified the more important and evident relations among these entities of our ordinary experiences, can be understood as an *ordering process giving rise to a space*. And since, apparently, most of our practical experiences were with classical entities, the *space of relations* that emerged from such penetration in width is what we call the *three-dimensional Euclidean space*. In other terms, *space* can be essentially considered as *a very specific theater of reality that emerged when a given set of experiences was properly ordered and organized* (i.e., put in relation to each other). In this view, the three-dimensional Euclidean space should not be imagined as an external and immutable container accommodating the different possible classical entities (following Newton's substantivalist view), but really as the manifestation of a *structure emerging from their relations* (in agreement with Leibniz's relationism), which in turn also depends on the very specific "way of interacting" we have focused on, in our initial process of penetration.

The reason why a specific *theater of reality*, which we have *constructed* during our long process of penetration first in depth and then in width of reality, has been mistaken, over time, for a fundamental substantive container for the latter (a position still maintained today by the majority of scientists) is easy to understand: as time went by, we have simply forgot about our construction, and since the typology of our experiences remained essentially the same, it was easy and natural to start believing that all of our reality should necessarily fit into such theater, so that the theater and its content, and reality, would just be one and the same thing. This belief, however, becomes difficult (if not impossible) to maintain when, for whatever reasons, new experiences are accessed, i.e., new entities, different from those that were discovered in our initial process of penetration, become suddenly available to be experienced, in a direct or indirect way, and because of their radically different nature, they do not let themselves be included in the relational space that we have built thus far. Also, if the construction of this relational space is such that no natural extensions of it can be conceived, that is, if the construction produced a sort of *closed environment*, in accordance with Heisenberg's notion of *closed theories* (Bokulich, 2008), then we will simply fail to incorporate these new entities and their new relations in the existing representation. Accordingly, they will be considered *non-spatial entities*, i.e., strange entities that although we cannot deny their existence, they nevertheless remain not permanently or fully representable (and therefore not permanently or fully present) in our Euclidean spatial "container."

Note that when we speak of new experiences, we do not necessarily mean that they must be new in a strict chronological sense. These experiences may actually be extremely old, if we consider when the corresponding elements of reality were available to us for the first time. However, what matters is if they were taken into consideration or not in our process of penetration, i.e., in the construction of an *intersubjective experiential space*, consensually shared by all human observers and participators of reality, or labeled as mere *anomalous experiences*, with no clear relation with the other experiences, so that they remained essentially without a dedicated place in the theater under construction, for instance because of their *ephemeral nature* (Sassoli de Bianchi, 2011), their *rarity*, or because not everybody was equally able to access them.

It may happen, however, that some of these non-ordinary experiences suddenly become more accessible, or obtain more attention, so that not only will there be an urgency to explain them, but also to find their mutual relations and, if possible, their relation with the entities that have already received a place within the current theater, i.e., with the entities that we usually associate with our ordinary experiences. Of course, the first attempt will always be that of trying to represent these new *non-ordinary entities* in the already existing representation, which in our discussion we have identified with the three-dimensional Euclidean space. However, as we mentioned, this attempt may turn out to be unsuccessful, because the nature of these non-ordinary entities may be too different to allow their inclusion into it, or even in an extended version of it. As we are going to explain, two paradigmatic examples of entities that have spoiled our efforts to incorporate them into our classical theater are the 'human conceptual entities', the 'quantum entities' and the 'relativistic entities'. This, probably, is not a coincidence, but a consequence of the fact that quantum (and in part also relativistic) entities, when viewed from a certain perspective, reveal a *conceptual nature* (although a non-human one).

2 Entanglement

As a paradigmatic example of a breakdown of our Euclidean theater construction, we can consider the discovery of so-called *entangled states*. At the theoretical level, they were initially discussed by *Einstein*, *Podolsky* and *Rosen* (1935), and by *Schrödinger* (1935), and their existence has now been firmly established in many experiments, for instance in the historical ones performed by *Alain Aspect* et al. with photons in 1982 (Aspect et al, 1982, 1999). The reason why entanglement is incompatible with our Euclidean construction is very simple to understand. As we said, during our 'penetration in width' of reality we have constructed a spatial representation of the different possible relations between the entities that we could identify, by means of our 'penetration in depth'. In this representation, the notion of *spatial distance* has been used to quantify the *separation* between the different entities, where the term 'separation' must be understood in the *experimental* sense. The idea is that the better an entity X is *experimentally separated* with respect to another entity Y, the greater is their Euclidean spatial distance $d_e(X,Y)$.

Being *experimentally separated* means that when we test a property on entity X, the outcome of the test will not depend on other tests we may perform (simultaneously or in different moments) on entity Y, and vice versa.[1] For ordinary (classical) entities this is guaranteed if the spatial distance separating them, and the time interval between the different tests, is such that no signal can propagate in time between the two entities, to possibly influence the outcomes of the respective tests; and in the limit where the two tests are performed in a perfectly simultaneous way, any finite distance is in principle sufficient to guarantee that we are in a *non-signaling condition*, i.e., in a situation of full experimental separation.

So, 'spatial separation' and 'experimental separation' were considered to be synonyms, as the former was precisely used to represent the latter, during the construction of our Euclidean theater. As an example, consider two objects, A and B, moving in opposite directions, and two experimenters jointly measure their respective positions and velocities. In general, there will be no *correlations* between the outcomes of their measurements, because the two entities are spatially separated, and therefore perfectly *disconnected*. This is a necessary but not sufficient condition, as there are situations where even though two entities are experimentally separated, i.e., disconnected, their properties can nevertheless be correlated. For this, it is sufficient that the two entities were connected in the past, and have been disconnected by some physical process, in such a way that the process of disconnection *created correlations*.

Take a rock initially at rest, say at the origin of the laboratory's system of coordinates, and assume that at some moment t_0 it explodes into two fragments A and B, of equal masses, flying apart in space (see Figure 1). The positions and velocities of these two fragments of rock will then be perfectly correlated, due to the conservation of momentum, i.e., if the position and velocity of fragment A, at a given subsequent instant t_1, are x and v, then the position and velocity of fragment B, at the same instant t_1, will be $-x$ and $-v$. Such perfect correlation, however, does not describe a situation of a persisting *interconnection* between the two fragments, being a simple consequence of the fact that the two fragments were previously part of a single whole entity.

[1] It will not depend on them in an ontological sense, rather, possibly, in a dynamical sense, for instance because both entities may interact by means of a force field, such as gravitational or electromagnetic fields.

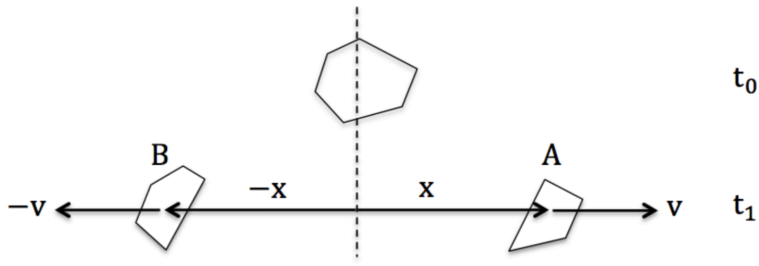

Figure 1 A rock initially at rest, explodes into two fragments of equal masses, flying apart in space.

In other terms, we have to distinguish between correlations that are already present in a bipartite system resulting from previous processes of connections-disconnection, which therefore can only be *discovered* during our observations, from correlations that are literally *created at the moment of their observation*, i.e., which are created out of an *actual connection* between the two parts of the bipartite system when these two parts are subjected to a measurement. This distinction is fundamental, and so it was proposed in 1990 to name the correlations that are only *discovered* in a measurement, '*correlations of the first kind*,' and those that instead are *created* in a measurement, '*correlations of the second kind*' (Aerts, 1990).

The important role played by the famous *Bell inequalities* (Bell, 1964, 1971), among other things, is precisely that of allowing one to distinguish between correlations of the first kind and of the second kind. Indeed, only the latter can violate Bell's inequalities (Sassoli de Bianchi, 2013a). However, contrary to the misconception that is still widespread among physicists, the violation of Bell's inequalities is not a specificity of quantum systems, as also classical macroscopic systems can easily violate them. This is so because, as we said, what truly matters for their violation is to deal with correlations of the second kind, created during the very process of measurement. But to create correlations one only needs the two entities forming the bipartite system to be *connected* in some way, and for classical (ordinary, macroscopic) entities a connection can easily be realized by creating a contact between the two entities, which can be direct or mediated by a third entity. As an example, imagine two identical dice connected through space by means of a rigid rod whose two ends are glued to the center of the dice's opposing faces. It is then easy to show that, because of the presence of the connecting rod, the double-die system becomes entangled, and correlations can be created in specifically designed coincidence "rolling measurements," able to maximally violate Bell's inequalities (Sassoli de Bianchi, 2013b, 2014).

Why, then, did Einstein call the quantum correlations produced by entangled entities, "spooky actions at a distance," when classical entities can easily produce quantum-like correlations, i.e., correlations of the second kind, violating Bell's inequalities? The answer is straightforward and brings us back to our discussion of the intrinsic limits of our Euclidean theater. A rod connecting two dice is clearly an element of reality that we can still represent within our three-dimensional Euclidean space, but the element of reality that establishes a connection between two microscopic quantum entities (like two photons, two electrons, etc.) in an entangled state cannot. So, the "spookiness" of the quantum correlations comes from the fact that: (1) they are not of the first kind and (2) the connecting element out of which they are created cannot be represented in our Euclidean theater. To describe this puzzling situation, physicists have used the term *non-locality*, but we think such a term is unfortunate, as it hides the essential nature of the phenomenon, which is the consequence of the existence of *non-spatial connections*, i.e., connections 'not happening in space'.

3 The EPR paradox

If it is correct to say that the observed correlations between entangled microscopic entities emerge from the existence of non-spatial connections, is it possible to extend the construction of our classical theater, for instance by adding more dimensions to it, to also include these new pure quantum elements of reality, which thanks to our modern laboratory experiments have recently discovered? Also, can we say that the quantum formalism has provided us with a more precise and encompassing description of the different entities forming our reality and their relations? Could we replace the classical Euclidean theater with some form of *quantum theater*, to represent all possible physical entities, their properties and mutual relations? Furthermore, can we incorporate the classical description into the quantum one? Of course, the answer to these questions will depend on how exactly we understand the terms 'quantum' and 'classical'. But let us give an example of what we think is a fundamental difference between the classical and the (standard) quantum formalisms, which make their associated 'theaters of reality' in a sense incommensurable. This example comes from the well-known (but not for this, we think, fully appreciated) situation called the *EPR paradox*.

In 1935, *Einstein* and his two collaborators, *Podolsky* and *Rosen* (a triumvirate we will designate by the acronym EPR) devised a very subtle *thought experiment* to highlight a possible inadequacy of the quantum mechanical formalism, in the description of the physical reality (Einstein et al, 1935). The reason their thought experiment was qualified as a paradox is that the predictions of quantum theory, regarding the outcomes of their proposed experiment, differed from those obtained when reasoning according to a very general *reality criterion*. If asked about the EPR paradox, most physicists will say that it has been solved by the mentioned *coincidence experiments* conducted on pair of entangled photons in singlet states, realized by *Alain Aspect* and his group in 1982 (Aspect et al, 1982), and which have been reproduced by many authors, with different quantum entities, under always better controlled experimental situations (Aspect, 1999; Hensen et al, 2015). Additionally, most physicists will say that these experiments contradict EPR's reasoning, in the sense that they confirmed the exactness of the quantum mechanical predictions. And this, of course, has considerably strengthened the general confidence of physicists regarding the completeness of the standard quantum framework, along with the fact that quantum mechanics would be the ultimate theoretical ambit within which all descriptions, laws, and experimental operations would need to be formulated in.

This conclusion, however, is the fruit of a misconception regarding the true nature of the EPR paradox, which had actually not been solved by experiments like those conducted by Aspect, but which can be solved by means of a simple, although quite subtle, mathematical reasoning (Aerts, 1981, 1984). This solution, however, says exactly the opposite of what is generally believed to be true: *the quantum mechanical description of reality is incomplete*; *we cannot straightforwardly incorporate a classical theater into a quantum one*, at least not if quantum theory is understood in its conventional sense. This also means that EPR were right in considering quantum theory incomplete, although the reason for its inadequacy was not what they thought. Now, since this result remains, in our view, exceedingly unknown, in the present and subsequent sections we will briefly explain its logic in the simplest possible terms, as this will also strengthen our proposal of putting forward a new view of realism, which we recently proposed to call *multiplex realism* (Aerts & Sassoli de Bianchi, 2016a), which consolidates what we currently know about the complex and multidimensional structure of our physical reality.

Let us start by reviewing what EPR did in their celebrated article (Einstein et al, 1935). First, they introduced the important notion of *element of reality*, by means of the following definition: "If without in any way disturbing the state of a physical entity the outcome of a certain observable can be predicted with certainty, there exists an element of reality corresponding to this outcome and this observable." This is one subtlety in the whole EPR reasoning which contains a very deep insight into the nature of reality: *that something real is there if one can predict it through an experiment which can be executed without disturbing the state of the entity in question*. Then, EPR considered the situation of two quantum entities that interacted and subsequently flew apart in space, thus becoming spatially separated and, according to EPR, also experimentally separated. The third step taken by EPR in their paper is to consider the quantum mechanical description of this situation, which they calculate in explicit terms, in accordance with the notion of entangled states, from which it can be seen that the positions and velocities are necessarily correlated, in the sense that, as was the case for the two aforementioned rock fragments, if one of the entities is observed to have position x, the other will certainly have position $-x$ (taking always the origin of the system of coordinates as the place where the entities interacted before flying apart), and if one of the entities is observed to have velocity v, the other one must certainly have momentum $-v$.

Now comes the truly subtle part of the EPR reasoning: they considered the situation where one could eventually measure the position of one of the quantum entities, let us say entity A, flying to the right (entity B then flying to the left). Suppose that such a measurement of position was carried out, and the position of entity A would be registered, for example as x. Then, according to the quantum description, it follows that the position of entity B can be predicted with certainty and is $-x$. Overall, this means that a measurement can be performed (the measurement of the position of entity B) that does not disturb the state of entity B (since the two entities are spatially separated, and therefore are also assumed to be experimentally separated) and predicts the position of entity B (position $-x$).

The same reasoning can be made for the velocity (or momentum), measured on entity A, which we will assume was found to be v. Similarly to the case of measuring position, the quantum description predicts that the velocity of entity B must then be $-v$, which means that again a measurement can be made that does not disturb the state of entity B and makes it possible to predict with certainty the value $-v$ for its velocity. But this means that both values of position and velocity can be predicted for entity B by means of measurements that do not disturb its state. Consequently, according to the aforementioned and very general reality criterion, both position and velocity can be said to have simultaneous definite values ($-x$ and $-v$ in the situation considered), which is clearly in contradiction with *Heisenberg's uncertainty relations*. Hence the paradox, and the EPR conclusion that quantum mechanics is an incomplete theory, as it cannot represent all the genuine elements of reality of a physical entity.

As is known, Bohr's reaction to the EPR argument was quite obscure. He said that one "is not allowed in quantum mechanics to make the type of reasoning proposed by EPR, and more specifically, the notion of element of reality does not make sense for quantum mechanical entities" (Bohr, 1935). The authority of Bohr and the general influence of the Copenhagen interpretation resulted in the majority of leading quantum physicists (with the notable exception of *Schrödinger*) accepting that there was not really a deep problem involved in the EPR paradox. Later, perhaps under the influence of *David Bohm*, who certainly took the EPR argument seriously (and invented the entangled spin example as a new and more transparent description of the EPR-like situation), a small group of physicists, among whom was *John Bell*, believed that EPR highlighted a fundamental problem in quantum mechanics related to its possible incompleteness. However, different from what EPR, Bohm, Bell and others believed, the incompleteness in question was not an issue of 'providing additional variables' to make it complete, or more complete, but a question of a much more severe shortcoming, related to the *impossibility for the quantum formalism to describe experimentally separated entities*.

4 The solution of the EPR paradox

To explain why this is so, let us start by emphasizing a point in the EPR reasoning that is usually overlooked: their reasoning is an *ex absurdum* one. Indeed, what EPR proved is that 'if quantum mechanics is a correct and complete theory', then it follows that 'it is an incomplete theory'. This is so because in their analysis they use quantum mechanics to describe the situation of two *separated* quantum entities A and B, flying apart following their interaction, assuming that the quantum description would be correct and complete, in the sense that it would accurately describe such situation. Now, even if the reasoning is *ex absurdum*, it is definitely possible to draw a conclusion from it when a contradiction is reached, and the conclusion is always that one of the premises of the reasoning must be false. So, by proving that if 'quantum mechanics is correct and complete' then 'quantum mechanics is incomplete', one can conclude that either (1) 'quantum mechanics is incorrect', or (2) 'quantum mechanics is incomplete'. Suppose for a moment that we exclude the possibility that quantum mechanics would be incorrect, as this was not at stake at the time of the EPR paradox, then the only possible conclusion is that quantum mechanics would be incomplete, and that is the reason why one can claim that the EPR reasoning proves, under the hypothesis that quantum mechanics is correct, that quantum mechanics is incomplete.

But, as we said, the status of this conclusion is that of a proof *ex absurdum*, and we know that we cannot attach any truth-value to the intermediate steps of a proof *ex absurdum*. This means that the step in the EPR reasoning saying that quantum mechanics has to be supplemented with additional variables has per se no truth-value at all. In other terms, it is incorrect to say that quantum mechanics is incomplete *and* needs to be supplemented with additional variables to solve its incompleteness. Indeed, the EPR reasoning does not offer any hint about what the nature of the incompleteness that it reveals would be. This is a point that has been overlooked by the majority of scientists studying the EPR paper, just as EPR were probably not aware of the '*ex absurdum* status' of their incompleteness proof. The forgoing reasoning makes it easy to understand why one of us, when trying some decades ago to elaborate a mathematical framework for the general description of separated quantum entities, was able to view the EPR work from a completely new angle. Indeed, while describing the situation of *bipartite systems formed by two separated quantum entities*, it was possible to prove, this time in a constructive way (not by a reasoning *ex absurdum*), that quantum mechanics was unable to describe this very simple situation.

This can be interpreted as an incompleteness of quantum mechanics (or incorrectness) and hence, it illuminates the origin of the contradiction identified by EPR in their reasoning. The situation is that: *quantum mechanics is an incomplete theory as it cannot describe the situation of separated quantum entities*, and since in the EPR reasoning quantum mechanics is used to describe a situation of quantum entities having interacted at a certain moment and flown apart, *which are then assumed to behave as separated quantum entities*, this necessarily leads to a contradiction. At first sight, as we explained, the contradiction identified in the EPR reasoning may suggest that one would need to introduce additional variables to allow position and velocity to have simultaneous definite values and escape the limitation of Heisenberg's uncertainty relations. But, a more careful analysis of the reasoning shows that this conclusion is incorrect, as the *ex-absurdum* reasoning does not allow one to deduce that that the introduction of additional variables (associated with the states of the two entities under investigation) would remedy the incompleteness.

Now, one could say: "All right, I follow your argument, and the conclusion that the nature of the incompleteness of quantum mechanics, already touched by the reasoning *ex absurdum* in the original EPR paper, would be the impossibility for the standard quantum formalism to describe separated quantum entities. But, have not EPR-like experiments, like those performed by the group of *Alain Aspect*, precisely shown that, in the situation considered by EPR, quantum mechanics does actually provide the correct description of two quantum entities flying apart, after having interacted? And since the experiments have shown that Bell's inequalities are violated, in accordance with the quantum predictions, doesn't this mean that quantum theory does actually correctly model the situation?"

The answer is both yes and no, hence some additional explanations are needed. At the time that EPR proposed the example of two quantum entities that have interacted and then flown apart, it was quite natural to expect that their spatial separation was equivalent to their experimental separation, i.e., that two entities, being spatially separated, were by definition also disconnected, i.e., experimentally separated (in accordance with the logic of our ancestral 'penetration in width' process). This expectation, however, has been overruled by many experiments, showing that, by making a big effort and by taking all sorts of precautions, one can indeed create experimental situations where microscopic entities, after having interacted, can remain interconnected (i.e., experimentally non-separated, entangled) even though arbitrarily large spatial distances separate them.

The possibility of producing these non-local/non-spatial states is remarkable and was totally unexpected, as it wasn't part of our previous construction of the Euclidean theater. It is however important to understand that it has very little to do with the EPR reasoning. Indeed, EPR, assumed that two quantum entities, when flying apart following their interaction, end up separated instead of remaining interconnected, as their *ex absurdum* reasoning can only be applied on such separated entities. Now, without diminishing the importance of the discovery of non-local/entangled states which open a window to a larger non-spatial reality impossible to fit into our three-dimensional theater, one has to understand that, in principle, experiments could also be done where instead of making a big effort to preserve the entanglement (i.e., the connection) when the two entities fly apart, an effort could be made to obtain the opposite situation: where two entities become disentangled when they fly apart.

Experiments of this kind have never been made consciously, because nobody realized that these would be the situations leading to the EPR paradox, i.e., that the incompleteness of quantum mechanics is not revealed in the physical situation of quantum entities flying apart and remaining non-separated, as these are the situations which are well described by the quantum formalism (as the violation of Bell's inequalities unequivocally proves), and there is no contradiction (no paradox) in this case. However, most of the EPR experiments that are usually interpreted as 'badly performed experiments' are certainly able to produce a disconnection in quantum entities flying apart, and these are precisely the situations that quantum mechanics is unable to describe.[2]

5 Multiplex realism

The experimental observation of quantum entangled states (by means of the violation of Bell's inequalities), and our logical analysis of the content of the EPR paradox, tell us two important things about our construction of a stage for the physical entities populating our reality. First of all, quantum entanglement demonstrates that they can remain interconnected even though arbitrarily large spatial distances separate them. This means that quantum entanglement cannot be fully represented in our Euclidean space, precisely because the reality of the connection it subtends cannot be associated with anything 'existing in space between the two entangled entities', i.e., with anything that would 'unite them through space'. In other terms, the classical Euclidean theater is too limited to stage the whole of the physical reality that we have been able to detect so far, by means of our instruments.

As we have explained already, we can understand this inadequacy by considering how special our human 'intraphysical condition' is, resulting from our manifesting, as consciousnesses, through our macroscopic and relatively dense vehicle of manifestation; a situation that has for instance been suggestively described by Plato in his famous *allegory of the cave*. The question, then, is: considering the newly discovered "quantum cave," as described by the mathematical structure of quantum mechanics, is it a more encompassing realm, containing in its interior the classical one, or is it, instead, just another cave, one certainly not directly inhabited by us humans, but one that is also the expression of a specific and necessarily also limited perspective on reality?

If we take seriously the content of the EPR paradox, and its solution, we see that the quantum cave also has its shortcomings. Indeed,

[2] Technically speaking, the reason for this is that there are properties of a system formed by two separated quantum entities that cannot be represented by orthogonal projection operators; see Aerts (1981, 1984).

in the same way as quantum entanglement cannot be represented in the classical cave (or theater, we shall use the terms 'cave' and 'theater' as synonyms), experimentally separated entities are also impossible to be described in the quantum cave. In other words, in the hypothetical world described by quantum theory, everything is inextricably interconnected and non-separable, but according to our experience with ordinary macroscopic entities, we definitely know that separation is also a possibility; it is, in fact, what was considered to be the rule before the advent of quantum physics (a rule that is incorporated in our construction of the Euclidean space).

So, if we want to remain prudent, all we can say so far is that different caves/theaters possibly exist, associated with the different vantage points that can be adopted on reality, and that in a sense reality is a construction about different possible representations. In other terms, as investigators/participators of reality, our work would not be only that of identifying the content of the theater we inhabit (and have created), and of the other possible theaters, but also the relations (the *partial morphisms*) existing between their different elements of reality, which we cannot *a priori* expect to be all representable/contained in a single and all-encompassing stage (although this remains a possibility we cannot logically exclude), as elements of reality belonging to one theater may not always find their equivalent in another one.

Most physicists still believe today that the quantum theater does contain the classical one, but as we explained we can cast serious doubts on that, as quantum mechanics cannot describe entities that are separated in experimental terms, whereas we can certainly experience plenty of separated entities in our classical theater. Also, there are elements of reality that appear to be of a genuine intermediate nature, like for instance those describing some of our human cognitive processes (more will be said about them later in the article), which cannot be conveniently represented neither in the classical nor in the pure quantum theater, as they seems to belong to a truly intermediate representation (Aerts & Sassoli de Bianchi, 2015a,b). Also, the failed tentative to unify gravitational and quantum elements of reality, within a unique, bigger and coherent representation, could very well be due to the fact that, for structural reasons, a single 'quantum plus gravitational' theater may simply not be possible to construct.

We have called the view we are putting forward in this article *multiplex realism* (Aerts & Sassoli de Bianchi, 2016a). This view gives due importance to the fact that we have been present for hundreds of thousands of years on the surface of our planet Earth, surrounded by material objects obeying with good approximation Newton's laws, that we have consequently staged in a Euclidean theater that we have expressly constructed to suitably describe our relations and interactions with these classical entities (which we are not saying are necessarily 'intrinsically classical', but that they certainly behave classically if only certain 'ways of interacting with them' are considered). Multiplex realism also considers that, in more recent times, we became aware of the existence of quantum and relativistic entities, whose reality could only be put in a *partial correspondence* with the properties and behaviors of the previously known Euclidean entities, i.e., that only *partial morphisms* could be found, and not perfect *isomorphisms*. Finally, multiplex realism emphasizes that if we want to deeply understand the more recently discovered non-Euclidian quantum and relativistic elements of reality, we need to become more aware of the historical 'construction aspect' of our Euclidean theater, and how it can affect our general understanding and conception of reality.

On the other hand, multiplex realism doesn't state that reality is necessarily multiple: realism can in principle be multiplex even if reality is fundamentally *singleplex*, in the sense that multiplex realism is an approach to reality that considers that certain incompatibilities, such as between relativity and quantum theories, can be more fruitfully studied if one considers the fact that their elements of reality are also, in part, the result of a constructive process. As a simple example, think of the Ptolemaic and Copernican worldviews, which consider different elements in their description of the *Solar system*, for instance as regards the Earth-Sun relative movement, depicted as the Sun moving on a circular (oblique) *ecliptic* inside the celestial sphere in the Ptolemaic description, and as the Earth moving on an elliptic orbit around the fixed Sun in the Copernican description. These different elements, however, are isomorphically linked, and thus can be understood as describing a same single reality: that of the Solar system. But as we will see, it is not clear anymore if 'multiplex realism' can be reduced, even in principle, to 'singleplex realism', when *aspects of creation* are involved in our observational processes, as appears to be the case when we consider the connection between the classical and quantum theaters.

6 The spin example

In our previous discussion, we have considered the notion of spatial separation and suggested that it was used, in our remote past, as a way to represent conditions of experimental separation, and that different 'degrees of separation' can be described by considering varying spatial distances between the objects (although we don't want to imply that this was the only motivation to introduce spatial distances). Of course, to describe all possible relations between the different macroscopic entities, additional spatial notions were also introduced, in particular the notion of *spatial direction*. However, in the same way that the classical notion of *spatial distance* has proven to be insufficient in accounting for all possible 'conditions of separation' between physical entities, we can ask if the notion of *direction* within a three-dimensional space would be insufficient to represent all possible *relative orientations* between physical entities.

Let us consider the paradigmatic example of the *spin* of a quantum entity, say of an atom, or of a molecule. When initially discovered, spin was understood as an intrinsic angular momentum carried by the microscopic entities. However, it was soon realized that it wasn't possible to associate it with a specific rotation in space. For instance, because if one tries to describe it in this way, one has to consider a superluminal velocity along the microscopic entity's periphery (when understood as a classical particle), in violation of the relativistic limit. Also, a spin, in general, cannot be represented as a *three-dimensional vector*, pointing in some direction, as is the case for the *angular momentum* of a macroscopic object. This is because in quantum mechanics a spin (and more generally an angular momentum) is described by an *operator* (defining a so-called *observable*), which cannot be simply drawn as a three-dimensional vector quantity whose components would be real numbers.

Can we however represent spin states as vector quantities, with real components? In other terms, can we construct a quantum theater in which the spin states of a quantum entity can still be regarded as directions in that space? By constructing a very general mathematical representation, called the *extended Bloch representation* (Aerts & Sassoli de Bianchi, 2014), this can indeed be done. For instance, if the spin in question is of magnitude s (according to quantum mechanics, s can only take integer or half-integer values), then the 'space of directions' that one needs to consider will have to possess $4s(s+1)$ dimensions. Thus, apart the situation of a so-called 'spin one-half entity' ($s=1/2$), which can still be described in a three-dimensional space, we see that the (Blochean) quantum theater, even when used to describe the orientation of a simple spin entity, will generally and necessarily be a space with more than three dimensions. But then, what is the relation between the directions available in the quantum theater, specifying the different possible spin states, and those available in our ordinary physical space? To answer this question, one has to start by observing that two different typologies of spin states need to be distinguished: the so-called *eigenstates* and *superposition states*. Eigenstates are by definition those states such that a space direction exists such that, if the value of the spin is measured along that direction (by means of a suitable apparatus, like a Stern-Gerlach one), the result of the measurement is certain in advance. According to EPR's reality's criterion, we can then say that eigenstates are associated with elements of reality that can be tested as from our Euclidean theater. On the other hand, superposition states are such that, whatever spatial direction is chosen for the measurement, the outcome can only be predicted in probabilistic terms, i.e., never with certainty.[3]

The difficulty in clarifying the origin of the quantum probabilities, associated with the measurement of superposition states, is generally referred to as the *measurement problem* (more will be said about it in the following). What is important here to observe is that superposition states describe conditions of the spin entity that are very different from anything related to a rotation along a spatial axis.

[3] Strictly speaking, a 'superposition state' is always relative to a given measurement. Indeed, a superposition state for a measurement can be an eigenstate for another measurement. For the purpose of our discussion, we are here using the term 'superposition state' in a more stringent way, to designate those states that can *never* be eigenstates, of whatever spin measurement (states of this kind only exist for entities whose spin is greater than one-half).

Thus, one would expect them to be genuine new elements of reality, only present in the quantum theater and totally absent from the classical one. But one would also expect the eigenstates to be, instead, those states that are still associable with specific spatial directions. This is in part certainly true, as is clear that a spin eigenstate, by definition, is always relative to a given spatial direction. However, it is not true in a general sense, as no spin states, be them superposition states or eigenstates, ever really points towards a spatial direction, within the quantum theater.

Let us try to explain the content of this last statement. If we interpret a spin as a classical 'state of rotation', then, it can be represented in our Euclidean theater by a vector of a given length, pointing toward a given direction. The length of the vector describes the value of the angular momentum (proportional to the value of the angular velocity times the moment of inertia) and the direction of the vector describes the axis of rotation. Thus, in our Euclidean theater, each 'state of rotation' is in a correspondence with a spatial direction. The situation is however different in the quantum (Blochean) theater. Indeed, it is possible to identify, within the higher-dimensional quantum theater, the subspace representing the ordinary spatial directions, i.e., those directions that are in a one-to-one correspondence with the three-dimensional Euclidean vectors, like those specifying the possible spatial orientations of the measuring apparatus. When this is done, one then discovers that none of the vectors describing the spin states in the quantum theater is ever aligned along a spatial direction, not even the eigenstates (Aerts & Sassoli de Bianchi, 2016a).

In other terms, we can say that spin entities are generally completely 'outside of space', in the sense that they are always pointing towards non-spatial directions. Thus, the spin of quantum entities provides further evidence in favor of our view of multiplex realism, i.e., that we need more than a single theater (space) to fully represent our reality. Note that this breakdown of the spatial description, in relation to spin entities, does not appear immediately for the special case of spin one-half entities, as for them the quantum Blochean theater is also three-dimensional; but as soon as two spin one-half entities are combined, one obtains a composite systems whose states already inhabit in a 15-dimensional space.[4] Now, when classical entities are combined, their angular momentum is just the sum of the angular momentum of each classical entity, from which it follows that the angular momentum of a composite system is still described by a three-dimensional vector. *But no sum of two three-dimensional vectors will ever be able to account for the emergence of a 15-dimensional vector space.* In other terms, there cannot be any simple relation between the spin-like elements of reality described in the classical three-dimensional theater and those belonging to the quantum one.

7 The ineffectiveness of Plato-Abbott's allegories

The spin example allows us to mention another important aspect of multiplex realism: the nature of the relation between the classical and quantum representations. One could naively be tempted to believe that the main difference between the quantum and classical theaters is their dimension, in the sense that the description of classical entities would simply be retrieved by means of some kind of projection from the higher-dimensional quantum theater to the lower-dimensional classical one. This would be like the situation described by the Greek philosopher *Plato*, in his famous *allegory of the cave*, where the entities he considered to have a deeper reality would cast some kind of *shadow* onto the lower-dimensional "walls" of our humanly constructed representation. A similar allegory was also conveyed by *Abbott*, in his

[4] In quantum mechanics, the combination of entities is obtained via a mathematical procedure called *tensor product*. If the complex Hilbert (state) space of an entity A is of dimension N, and the Hilbert space of another entity B is of dimension M, then the Hilbert space of the entity obtained from their combination (via the tensor product) is of dimension N times M. If the two entities are spin one-half entities, then $N = M = 2$. Also, in the *extended Bloch representation of quantum mechanics*, the dimension of the (real) state space associated with an entity whose (complex) Hilbert space is of dimension N, is of dimension $N^2 - 1$. Thus, the Blochean state space of a spin one-half entity is of dimension $2^2 - 1 = 3$, and the Blochean state space of two spin one-half entities is of dimension $4^2 - 1 = 15$ (Aerts & Sassoli de Bianchi, 2014, 2016a).

famous 'Romance in Many Dimensions' (Abbott, 1884).

Our view of multiplex realism, like the views expressed by Plato and Abbott, affirms that our Euclidean theater, similarly to Plato's cave and Abbott's Flatland, is the expression of a limited perspective. But there is an important difference: according to multiplex realism we don't have a situation where an ultimate, more encompassing representation would necessarily exist. Instead, different "caves" are more likely to exist, or to be constructible, each one offering a different and unique vantage point on our reality. In that sense, multiplex realism is more similar to *Heisenberg's pluralistic view on realism*, expressed in his *closed theories* account (Bokulich, 2008). According to *Heisenberg*, quantum mechanics was not to be considered a more fundamental theory than classical mechanics (or other theories, like statistical thermodynamics or *Maxwell*'s electromagnetism together with optics and special relativity), in the sense that for him both theories were necessary to obtain a more complete description of reality, not only because each of them would have its specific domain of validity, but also because they would correspond to final and perfect descriptions of their domains. This is in part in agreement with our multiplex view, where we have assumed that different theaters of reality can be constructed to capture the properties and relations of certain 'domains of entities', and that once a specific representation is considered, it defines a 'relational space' which remains closed (in the sense that only certain physical relations can be described into it, and not others), similarly to how Heisenberg considered certain theories to be also closed.

However, in contrast to Heisenberg, in our approach, we link the multiplex appearance of our reality to the very particular (parochial) condition in which we humans find ourselves, for historical reasons, and more specifically to the fact that we penetrate reality: (1) from within a niche which is very particular, in the sense of only containing a particular type of entities and elements of reality; (2) by means of a very particular exploratory modality, because of the characteristics of our bodies, human minds, etc. This means that we do not a priori make any claim about the nature of reality in itself. Our only claim is about 'realism', i.e., the fact that if one starts to carefully and consciously collect different elements of reality, using all means available, then because of the limitations brought about by the two above aspects, the state of affairs that will generally result is that of 'multiplex realism', even if reality itself would be unique and well defined. Considering that realistic theories will inevitably result in a multiplex view, the possibility that reality itself might be multiplex remains a possibility.

Having said this, let us stress once more that there is not only the issue of being able to find only partial, instead of full, correspondences (isomorphisms). There is also the more important matter that we cannot generally expect that, when elements of reality belonging to a given representation (like the quantum one) are viewed from our classical representation, which includes our macroscopic measuring instruments, the viewing process would always be amenable to a mere *discovery process*. This, as we know, is not the case when dealing with so-called superposition states, for which the process of measurement is not a mere observation of pre-existing properties, but of creations of properties that were non-existent prior to the measurement. In other terms, when different theaters, or caves, are put in relation with one another, there is an additional difficulty to take into consideration, which was not envisaged by Plato or Abbott in their thought-provoking allegories: when an entity that is not contained in our Euclidean theater is observed from the perspective of the latter, we do not just *discover* the lower-dimensional shadow of that higher-dimensional entity. Indeed, if the entity is in a superposition state (with respect to the measurement in question), then the outcome of the observation will not be just something to be *discovered*: an irreducible *creation* aspect will also be involved, in the sense that the value of the observed (i.e., measured) physical quantity will not be given in advance, but will be literally created by the very process of observation.

This *non-deterministic* process of creation of an outcome, resulting from the observation of 'ordinary elements of reality' on 'non-ordinary entities', like quantum entities, can be explained as a process of *weighted symmetry breaking* (Aerts & Sassoli de Bianchi, 2016b). This explanation is supported by the previously mentioned extended Bloch representation, which allows for the association of a predetermined number of non-spatial *hidden-measurement interactions,* responsible for the actualization of the available outcomes, to each measurement. The relative number of interactions associated with an outcome then determines its probability (according to the quantum mechanical *Born rule*), and since nothing in the process favors one interaction with respect to another, it is impossible for the experimenter to *a priori* determine which one will be ultimately selected (following the inevitable fluctuations that are part of a quantum measurement context and that by no means can be controlled by the experimenter without dramatically altering the measurement itself), hence the irreducible indeterministic character of a quantum measurement; see for instance (Sassoli de Bianchi, 2015), in this journal, for a simple description of the hidden measurement interpretation of quantum mechanics.

What is important to emphasize is that those aspects that are not representable in the Euclidean theater (and, more generally, described by a classical theory) correspond to what we usually describe, from the perspective of the latter, as *potential elements of reality*, or *potential properties*, and a quantum measurement is nothing but a process where *the actual breaks the symmetry of the potential*. So, when considering the interplay between the different theaters, we have to consider that the encounter between entities belonging to different theaters (for instance an electron belonging to the quantum theater and a Stern-Gerlach apparatus belonging to the classical theater) will necessarily involve aspects of creation, as a measurement process will always *force* the measured entity to acquires those elements of reality (not previously possessed) which will allow it to *momentarily enter the stage in which the measurement is performed*. This "coming on stage," however, can only occur in a perfectly indeterministic way. In fact, and this is a subtle point to grasp, it is precisely because the process is genuinely indeterministic that we can actually speak of a process of creation, i.e., that we cannot associate in advance the property that is observed before it is observed. Indeed, if the process were deterministic, the outcome would be certain, and according to the EPR criterion it would be associable with an element of reality existing prior to the measurement.

8 Reinterpreting quantum experiments

As the examples of 'entanglement' and 'spin directions' clearly show, a quantum entity, be it single or composite, cannot in general be understood as an entity belonging to our three-dimensional spatial theater, nor as a higher-dimensional classical-like entity that would simply 'cast a three-dimensional shadow' onto it. In spite of that, quantum entities can certainly maintain a stable relation with the classical entities. In other terms, it is precisely because there is a correspondence, however partial, between the elements of reality that are present in the quantum (non-spatial) theater and those in the classical (spatial) theater that we have been able to discover all the strange properties of the former. But this needs not to be the case for all existing physical entities. To remain within the example of spins, the reason why we can measure them with instruments belonging to our Euclidean theater is because, although 'out of space', they nevertheless are always in a specific relation to space and its classical elements of reality. But, there are quantum properties, for instance the *color charge* of individual quarks, which, as far as we know, and contrary to spin values, have no evident relation to space, and this may in part explain why certain entities appear to remain confined within the "quantum cave." In that respect, our analysis puts forward a possible new way to explain why color charged elementary entities, such as quarks, cannot be singularly observed, which is different from the standard more phenomenological explanation.

Once the notion of non-spatiality becomes an integral part of our description of quantum entities, much of our difficulty in understanding their nature and behavior disappears. Of course, their behavior remains strange according to our standards, as is clear that they remain non-ordinary entities, i.e., entities that were not considered when we constructed our initial representation of reality, which is the representation we also considered when, in more recent (scientific) times, we formalized our knowledge in so-called classical (Newtonian) mechanics. We have mentioned already that the senses of sight and touch have played in this a primary role. Before continuing, let us briefly explain the reasons to believe that these two senses are precisely those that have mostly fooled us into the illusion that all physical entities would be objects (things) inside space (or even that, what we call objects, can only behave as such).

Our sight, however sophisticated, is a very classical instrument, and it is probably not a coincidence that it was reinvented in our human technology as *photography camera*. It makes use of what is called in physics the *geometrical theory of light*, where the latter is considered to be formed by infinitely thin rays always traveling in straight lines. This, however, is a highly simplified theory, which is a good approximation only at the very specific scale and frequencies our eyes operate. For instance, insects have a very different type of eye, and at their scale this geometrical approximation already fails, mainly due to *diffraction phenomena*, which become dominant if a *camera obscura* type of eye becomes as small as an insect eye, causing the vision to become completely blurred. So, by using our eyes, we explore a world that is quite illusory, in the sense that we can only properly capture a portion of it: that which is compatible with our specific scale and size.

Regarding touch, we tend to believe that it provides us with a very intimate, and hence, also very deep contact with reality. But, is it really so? In principle, touch is a possibility that is mainly a consequence of *Pauli's exclusion principle*, stating that *fermions* (the "particles" of matter, also forming our body) cannot collectively occupy the same states. Consequently, if we touch a chair with our fingers, since the electrons in the fingers cannot occupy the same state as the electrons in the chair, we 'feel' an emergent pressure (called *degeneracy pressure*), expressing the impossibility not only to penetrate the matter of the chair with the matter of our fingers, but also to compress the matter of the chair into smaller volumes of space. Now, touch is a sense that certainly works on a deeper level than sight, but is also governed by a simple principle that only applies to one typology of the known physical entities: *fermions*. For example, photons, the constituents of light, which are not fermions but *bosons*, can easily occupy all the same state, independently of their number. In fact, the more of them that are in a given state, the more probable it is that others will enter that same state, an aptitude that is exploited in our *laser technology*.

Now, although they work on very different physical principles, the two senses of sight and touch are perfectly compatible with one another, as is clear that we see the chair with our eyes exactly where we perceive it when we touch it, and this "synesthesia" (understood here as a "working together, in a compatible way") has clearly contributed enormously to the illusion of a three-dimensional world formed by macroscopic objects, and to the increasingly dominant role played by our sight and touch in deciding the nature of the world we live in, despite the fact that our other senses, and the more recent "interaction" guided by language and meaning, is in principle able to bring us closer to the deeper, non-spatial layer of our reality, which we discovered in our quantum laboratories.

Quantum experiments certainly ask us to abandon the illusion of a reality that would be fully contained in space. Entangled states, as we explained already, are the expression of connections extending beyond space, and spins cannot generally be depicted as rotations along any spatial axis. But let us give another example of paradigmatic experiments that remain totally unintelligible if one tries to interpret them using the "space contains reality" prejudice: *Wheeler's delayed choice experiments*. In experiments of this kind, which were first imagined by Wheeler (1978), a quantum entity enters a measuring apparatus like, say, the one used in a *double-slit experiment*, whose arrangement, however, can be changed at the last moment, before the entity (a photon, an electron, a neutron, etc.) is finally detected. Only two possible arrangements are considered: the first one, let us call it the 'wave arrangement', corresponds to the usual one adopted in a double slit experiment, producing the typical interference effects on the detection screen; in the second arrangement, let us call it the 'particle arrangement', the screen is removed and replaced by a pair of detectors, positioned in such a way that the statistics of their clicks now becomes fully compatible with a particle description (no more interference effects); see Figure 2.

The idea of a delayed choice experiment is to change the arrangement very rapidly, but only after the quantum entity has already passed through the two slits. Many authors have successfully performed these experiments, over the last decades, exploiting different techniques and properties of the quantum entities (like the possibility to correlate the path taken by a photon with its polarization). The results are always that, even though the change in arrangement happens at the very last moment, the entity behaves compatibly with it, i.e., as if the final arrangement was there since the very beginning.

The intention behind experiments of this kind is to have the quantum entity first pass through the double-slit region, either in the particle or wave arrangements, so that, according to what we may call the *wave-particle prejudice*, it will be "forced" to either behave as a particle or as a wave. Then, just before being detected, the arrangement is suddenly changed; more precisely, if the entity entered the double-slit region in the 'particle arrangement', thus, according to the prejudice, as a particle, the screen is abruptly placed in front of the pair of detectors (see Figure 2), and if the entity entered the double-slit region in the 'wave arrangement', thus, according to the prejudice, as a wave, the screen is suddenly removed, so that the pair of detectors is operational.

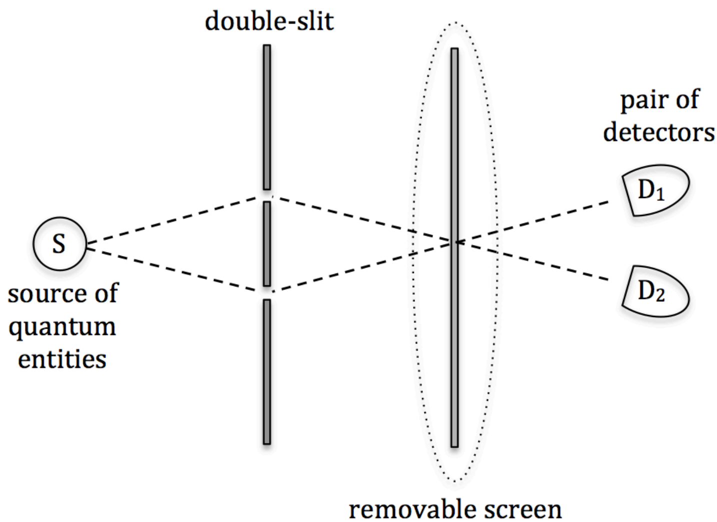

Figure 2 A schematic illustration of a delayed choice experiment. The screen is removable, and it may be either left in place (to create a 'wave context') or removed (to create a 'particle context').

Now, if we assume that the quantum entity can only be a wave or a particle, depending on the context (i.e., on the arrangement), then, once it has passed through the double-slit structure as a wave, interferences will have to take place, and it is reasonable to think that a subsequent change of the arrangement cannot make them disappear. Similarly, once it has passed through the double-slit structure as a particle, there should not be any interference effects, and again it is reasonable to consider that a subsequent change of the arrangement cannot retroactively create the interferences. But this is not what is observed when the experiments are carried out: everything always happens as if the quantum entity would have "delayed its choice" of manifesting either as a particle or as a wave, until the final arrangement of the detection apparatus is selected.

The wave-particle prejudice is already difficult to understand *per se*. How a quantum entity can transform into a particle, or a wave, depending on the context, is increasingly challenging to elucidate. But, even if this would be possible, to understand then the result of the experiments, one should also assume that the quantum entity would be equipped with some sort of precognitive abilities, knowing in advance what the final arrangement would be, and transforming accordingly either as a particle or as a wave. Or, if not capable of making predictions of the future, it should then be able to produce some kind of retro-causation. Another possibility is to adopt a radical antirealist stance, like the one of Wheeler who, facing the implications of his delayed-choice experiments, famously concluded that "no phenomenon is a phenomenon until it is an observed phenomenon,"[5] so that reality "out there" would not exist independent of our acts of observation.

But, instead of renouncing the existence of reality until it is measured, in total disagreement with the general EPR reality criterion, it is sufficient to consider that the quantum entity subjected to the experiment is neither a particle nor a wave, and in fact it is never one or the other, as particles and waves are spatial entities (particles are localized spatial entities and waves are spread out spatial entities) whereas a quantum entity, like a photon or an electron, generally behaves as a non-spatial entity. And, since the quantum entity retains its non-spatial condition for as long as it is not "sucked into space," it doesn't have to sense the apparatus in advance, nor to retroact, but just behave according to its non-spatial nature, until it is suddenly brought into space at the moment of the detection, by either interacting with the screen or with the pair of detectors. So, the quantum entity does not delay its choice of manifesting either a particle nature or a wave nature until it is detected, but is simply detected (in the present case, absorbed) in a way that depends on the *final measuring context*. The quantum entity perfectly exists prior to the detection, although it does so as a non-spatial entity, not having any specific spatial attributes, and when it is detected it manifests in a way that we can, retrospectively, interpret (following our wave-particle prejudice) as 'the effect of a particle' or 'the effect of

[5] Quoted in (Scully & Scully, 2007), p. 191.

a wave', depending on the final context.[6]

To explain this unusual behavior of a quantum entity only apparently delaying its choice of manifesting in a way instead of another, let us consider a metaphor that, as we will explain later, may be more than just that. We replace the quantum entity by a human concept and, to be more specific, by the concept 'Mouse' [this is an adaptation of an example taken from Buonomano (2011)]. The (apparent) passage of the quantum entity through the double slit apparatus is replaced in our metaphor by the concept 'Mouse' being combined with the concept 'On the table', producing the combined concept 'The mouse on the table'. On the other hand, the 'particle arrangement' and 'wave arrangement' that, according to the experimental protocol, are applied in a definitive manner only at the very end, are described in our conceptual analogy by the concepts 'Is broken' and 'Is squeaking', respectively. So, in the first case, we obtain the final combination 'The mouse on the table is broken', and in the second case the combination 'The mouse on the table is squeaking'. Finally, consider a human mind that is asked to further specify (concretize) either the concept 'The mouse on the table is broken', or 'The mouse on the table is squeaking', by picking one of the following two options: $\psi^A =$ 'The small rodent on the table is squeaking', or $\psi^B =$ 'The computer pointing device on the table is broken'. These two possible answers, in our example, correspond to a final concretization/spatialization of the abstract/non-spatial concept 'Mouse', and in our metaphor are meant to represent the two situations 'the impact on the screen is produced by a wave-like entity' and 'the click in the detector is produced by a particle-like entity', respectively.

The concept 'Mouse', in the beginning, is not embedded in any semantic context, so that its state can just be said to correspond to the "ground" state $\psi_0 =$ 'The mouse is a mouse'. Then, the context changes, and 'Mouse' gets combined with the concept 'On the table'.

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[6] Consider however that microscopic quantum entities are always individually detected as localized impacts, and that it is only when an entire statistics of impacts is analyzed that a wave-like or particle-like behavior can possibly be inferred. Also, this remains true only in the description of individual entities: when systems formed by more than one entity are considered, the wave-like patterns one can possibly observe can no longer be understood as resulting from the interference of three-dimensional waves.

One can describe this situation by saying that the concept state has evolved from the "ground" state ψ_0 to the "excited" state $\psi_1 =$ 'The mouse on the table' (i.e., the state obtained by putting the 'Mouse' concept in the context of the 'On the table' concept). Then, depending on the final choice of the "arrangement" by the experimenter, this ψ_1 state is either further evolved into the state $\psi_2^A =$ 'The mouse on the table is squeaking' or the state $\psi_2^B =$ 'The mouse on the table is broken'. At this point, a human mind will interact with one of these two states to "collapse" it, by choosing one of the two more specific descriptions ψ^A or ψ^B (outcome states).

Here we are in a situation where, with a probability very close to 1, a human mind will collapse state ψ_2^A to ψ^A (in the quantum jargon, one says that ψ_2^A is very close to the eigenstate ψ^A), and state ψ_2^B to ψ^B, but less deterministic contexts could also be considered. Let us now explain why this 'conceptual metaphor' can be helpful in elucidating what goes on behind the spatial scenes in a delayed choice experiment. We have to consider that a human mind selecting the final state (either ψ^A or ψ^B) has here the scope of attributing a final (more concrete) meaning to the concept 'Mouse', when the concept is presented to it in state ψ_2^A or ψ_2^B. However, we see that this meaning remains undetermined when the 'Mouse' concept is in state $\psi_1 =$ 'The mouse on the table', and that it is only when the final conceptual fragment is added (the equivalent of the final choice of an arrangement in the delayed-choice experiment), producing either state ψ_2^A or state ψ_2^B, that this meaning can become more evident.

In other terms, a more explicit meaning of 'Mouse' is only obtained when one of the two final contexts 'Is broken' and 'Is squeaking' is added. Before that, 'Mouse' remains, within the human conceptual reality, in a more abstract state, expressing a variety of potential more concrete meanings. But even more interesting is to consider what happens in time when the mind hears, say, the sentence 'The mouse on the table is broken'. Until the last word is heard, the mind will wait to attribute a final more concrete meaning to 'Mouse', and more importantly, this more concrete meaning will manifest suddenly, as a whole. In other words, although it is certainly correct to say that the mind *delays its choice*, regarding what would be a specific meaning for 'Mouse', once this choice is made it doesn't edit retroactively the entire sequence, with 'Mouse' now replaced by 'Computer pointing device'.

So, the 'Mouse' conceptual entity subjected to the cognitive experiment is neither a small rodent nor a computer pointer device, at least for as long it is not "sucked into a more concrete conceptual space," by interacting with a specific measurement context. And it would be meaningless to say that the 'Mouse' conceptual entity would be able to sense such specific context in advance, or retroact in some way: it simply has to behave according to its abstract nature, until it is suddenly concretized, by either interacting with the 'Is broken' context or the 'Is squeaking' context. The conceptual entity 'Mouse' does not delay its choice of manifesting either as a small rodent or as a computer pointer device: it simply exists as an abstract entity, having a number of potential meanings, until it is ultimately detected by a mind-like entity, sensitive to these potential meanings, which then selects one of them in a more or less deterministic way.

9 The conceptuality interpretation

Considering the previous metaphor, one may wonder if there would be something deeper in the analogy between quantum entities and human concepts. In other terms, one may wonder if (1) human concepts would behave similarly to quantum entities, i.e., if they have a quantum-like nature, and, *vice versa*, if (2) quantum entities would behave similarly to human concepts, i.e., if they have a conceptual-like nature. In our view, both points can be answered affirmatively: point (1) resulted (for about fifteen years now) in the emerging field of investigation known as *quantum cognition*; see for instance: Busemeyer & Bruza (2012), Haven & Khrennikov (2013), Wendt (2015), Aerts et al. (2013, 2016), Aerts & Sassoli de Bianchi (2017); point (2) resulted in a novel interpretation of quantum mechanics, known as the *conceptuality interpretation* (Aerts, 2009, 2010a,b, 2011).

Quantum cognition should not be confused with the theory of the quantum brain, speculating that quantum mechanical phenomena, at the micro level, may also play an important role in the brain's function, particularly in relation to the manifestation of the consciousness. It is about applying the conceptual and mathematical formalism of quantum theory to model different cognitive situations, for instance those involving decision-making, conceptual reasoning, human memory, and other cognitive phenomena. The reason for doing so is that, in the same way physicists were confronted with data that appeared to be inconsistent when viewed from the perspective of classical mechanics and classical probability theory bringing them to the development of quantum mechanics, psychologists and cognitive scientists were also confronted with anomalous (irrational) human behavior, if analyzed according to classical logic and classical probability theory. When these behaviors where organized in the ambit of statistical studies, conducted on significant samples of subjects, the only way to explain the structure of the obtained experimental probabilities was to resort to non-classical (non-Kolmogorovian) probability models, like the Hilbert model of quantum mechanics, or even more general (quantum-like) models, neither classical nor quantum, but somehow in-between.

In other terms, assuming that human thought and reasoning is formed of two layers, a *logical* one, of an *analytical* nature (usually and erroneously taken for granted), and a *conceptual* one, of a *synthetic* nature (usually and erroneously considered to be anomalous), the first layer can be efficaciously modeled by classical probability models, whereas the second layer can only be modeled by quantum (or more generally quantum-like) probability models. Thus, depending on the cognitive situations, our minds can function either as classical or quantum machines (and many times also both, simultaneously), thus requiring not only classical but also quantum notions to be properly modeled (Aerts & D'Hooghe, 2009).

It is certainly not our intention in this article to explain, even in broad terms, the different aspects of the particular field of research of 'quantum cognition' and the successes that have marked its rapid expansion over the last years (in particular, how it was possible to describe the highly *contextual* and *indeterministic* part of human cognitive processes by exploiting the quantum formalism and some of the most salient features of quantum systems, like *superposition*, *interferences*, *correlations due to entanglement* and even the 'many-body effects' typical of *quantum field theory)*. Let us however mention that different approaches exist, adopting different ontologies, and that the approach that was taken by the Brussels' group, since the dawn of this new field of study, was an *operational-realistic* one, where the different conceptual entities interacting with the human minds are considered to possess objective (i.e., intersubjective) properties, independent from the minds possibly interacting with them (Aerts et al., 2016).

More precisely, according to Brussels' operational-realistic approach to cognition, human concepts can be viewed as 'meaning entities' that can be in different states, depending on the context in which they are, as we suggested already in our previous 'Mouse' example. Concepts can combine, forming 'composite conceptual entities', and when they do so they can 'connect through meaning'. These meaning connections can create correlations (of the second kind), which in turn are able to violate Bell's inequalities, exactly in the same way quantum correlations can do. In other terms, entanglement systematically occurs in human cognition in a way that is similar to quantum entanglement (Aerts et al, 2000, 2011, 2018a,b).

Similarly, the states of human concepts can 'collapse' as the states of quantum entities do, transitioning from more *abstract* superposition states to more *concrete* eigenstates, during the measurement (interrogative) processes. If, in a 'physics laboratory', the collapse is produced by the interaction with a measuring apparatus, in a 'psychological laboratory' the collapse is produced by the interaction with a human mind, playing exactly the role of the measuring entity, when selecting one among the available outcomes, taking into consideration, for instance, their *representativeness*, or *typicality*. But as soon as one acknowledges the quite amazing analogies between human concepts and quantum entities, and between psychological measurements and quantum measurements, one is forced to also consider that the analogy works in both directions, i.e., that quantum entities may actually behave in such a strange way precisely because they would not be objects (like particles, corpuscles, bodies, waves, fields, etc.), but *conceptual entities* (although of a non-human kind) able to be in states of different degrees of abstractness, interacting with each other in a similar way as human concepts combine with each other, and interacting with the different measuring devices in a similar way as human concepts interact with the human minds.

10 The relativistic principle

We would like now to add a different example of entities that cannot be easily incorporated in our Euclidean theater: the relativistic ones. Usually, the term 'relativistic' refers to entities that would make manifest the so-called *relativistic effects*, as described in Einstein's special relativity theory, because they would move in space at speeds comparable to the speed of light. In the following, we will not consider quantum entities, in the sense that we will not assume that these *relativistic entities* would be subjected to indeterministic measurements, contextuality and emergence effects that are typical of quantum entities. We really consider them, for the moment, to be just classical bodies that can move in space.

The term 'relativity' has been historically attached to Einstein's work; however, it more exactly refers to a principle – the *principle of relativity* – which is much more ancient. It was described by *Galileo Galilei*, in his 1632 dialogue concerning the two chief world systems, with his famous example of the ship advancing at uniform speed, with people locked in the cabin beneath the deck of the ship not being able to determine (by observing different phenomena) whether the ship was moving or just standing still (Galilei, 1632). But one also finds descriptions of this principle as early as the first century B.C., that is, 1700 years before Galileo, in China. In the *Apocryphal Treatise on the Shang Shu Section of the Historical Classic: Investigation of the Mysterious Brightnesses* (*Shang Shu Wei Kao Ling Yao*), we can indeed read: "Although people don't know it, the earth is constantly moving, just as someone sitting in a large boat with the cabin window closed is unaware that the boat is moving."

In a nutshell, the relativistic principle can be enounced as follows: "Equivalent viewpoints exist on the physical world." When formalized using the more specific notion of *reference frame*, it then becomes [see for instance Lévy-Leblond (1977)]: "Equivalent frames of reference (space-time coordinate systems) exist for the physical laws, i.e., such that the physical laws have exactly the same form in all of them." This doesn't mean that the values of the different physical quantities will be the same in the different equivalent reference frames: it simply means that they will obey exact the same relations, which in turn means that phenomena will be observed exactly in the same way, when viewed from these different equivalent reference frames.

Of course, not all reference frames will be equivalent. When for instance we are on a carousel, which is rotating at a given speed, we will observe and experience phenomena that would be absent if the carousel would be at rest, such as the well-known centrifugal pseudo forces. So, the interesting content of the principle of relativity is that, among the infinite number of possible reference frames, some exist that are perfectly equivalent. Now, the simplest (and in a sense also trivial) examples of equivalent reference frames are those that are just translated, i.e., that differ from one another simply because they don't share the same origin for the spatial and temporal coordinates. Another possibility is that of reference frames whose axis would have different orientations. But Galileo, and before him the ancient Chinese sages, identified a more interesting and non-trivial class of equivalent reference frames: those *moving at a constant speed* with respect to the others, nowadays called *inertial frames*.

The fact that inertial frames are equivalent has some remarkable consequences: one is that an object moving at constant speed must be characterized, from the viewpoint of the physical laws, in exactly the same way as an object at rest, and such characterization is that of an absence of forces acting on the object. The so-called *principle of inertia* (also known as the *first law of Newton*) immediately follows: like an object at rest, an object in motion at constant speed will remain in such state of motion forever, unless acted upon by a force. But there is a more dramatic consequence of the relativity principle, which certainly hasn't been fully appreciated until Einstein's relativity came on the scene, many centuries after Galileo: if inertial frames are equivalent frames, then making sense of a notion of *absolute movement* is no longer straightforward.

This means that, already considering the Galilean principle of relativity, and long before the advent of Einstein's theories, our understanding (theorization) of space dramatically changed in comparison to our initial 'penetration in width' of the ordinary physical reality present on the crust of our planet. Indeed, if making sense of an 'absolute state of spatial movement' becomes convoluted, this has consequences for the possibility of making sense of the notion of *space as a fundamental substantive container*. In other words, the ancient Chinese sages, or Galileo, if they had given careful consideration to all consequences of their boat examples, they could have come to the conclusion that 'a notion of space, as a theater for the whole of our reality, is problematic'.

The principle of relativity, with the existence of the non-trivial equivalent inertial frames, tells us that space is most probably essentially a *relational construct*. And, if we continue along such a hypothesis, since each entity has then a unique perspective, it follows that *each physical entity actually inhabits a different space*, i.e., a different relational spatial structure. This becomes an almost trivial observation if one considers the so-called *parallax effects*, i.e., the fact that the different entities, because of their different viewpoints, will generally locate a singular object in different positions. Of course, one can immediately object, and rightly so, that when we combine all these different viewpoints, a 3-dimensional structure will emerge, which is precisely the 3-dimensional Euclidean space, perfectly explaining the parallax effects. In other words, it is precisely because parallax effects exist that we can say that we all inhabit a *same* space.

This is indeed correct, and in fact it is precisely what animals (like us) with a *binocular vision* have learned to do: to develop a 'perception of depth' (not to be confused with the notion of 'penetration in depth' introduced in this article), obtained from the different viewpoints of each of their two eyes (*stereopsis*). So, spatial parallax effects are certainly not sufficient *per se* to support the idea that each entity would construct an individual relational space. In fact, the idea of the existence of an objective "container space" is further reinforced by the fact that not only can we deduce this space by combining all these different perspectives, but also that *we can see the different entities moving into it*.

However, do we see entities moving in space because they actually move in a spatial theater, or is it just because we confer to them a spatial movement to "keep them inside our personal spatial representation"? This may look like a twisted question, trying to complicate things rather than simplify them. But the question is more than legitimate, considering that the principle of relativity tells us that it is not straightforward to make sense of an 'objective state of (spatial) movement'. If we can certainly all agree on the fact that an entity is present in space in some location (which will be described by different numbers in the different reference frames), it is no longer possible to agree on the fact that such entity would be 'moving in space', as for some observers it will be perceived to be at rest. Thus, if on one hand movement is what allows us to 'keep entities in space', it is also what is telling us that there cannot be a single all-encompassing spatial container for all the existing physical entities.

The situation becomes much more dramatic with the passage from Galileo to Einstein. The so-called *Galilean transformations*, used to transform between the coordinates of different Galilean inertial frames, only concern the transformation of the spatial coordinates, not the temporal ones. In other words, if two Galilean inertial observers will generally attach different coordinates and velocities to a same object, they will nevertheless describe time (i.e., the movement of their clocks, also called, although improperly, the *time flow*[7]) in exactly the same way. With the advent of Einsteinian relativity, these transformations are replaced by the more general *Lorentz transformation*.

The difference between the Galilean transformations and the Lorentz transformations resides in two important aspects. The first one is that in the Galilean case it was taken for granted that *standards of length* had to remain the same in the different inertial frames. In other words, it was assumed that the length of objects would remain the same when measured from different inertial frames. This assumption was clearly natural at the time of Galileo, as this is what was actually observed, and logical, considering the preconception of living entirely in a three-dimensional theater. But when motions at speeds that are no longer negligible with respect to that of light were considered, a different scenario was revealed: objects, when moving, are measured to be shorter in comparison to when they are at rest.

The second aspect distinguishing the Galilean from the Lorentz

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[7] We don't have to commit the mistake of confusing something with its function. When we say that 'time flows', it is a bit like saying that a 'walkway walks'; a walkway cannot walk: a walkway is what allows people to walk. Similarly, time is what allows 'reality to flow', and therefore it cannot itself flow. So, when we improperly speak about the flow of time, what we have to understand is that we are really talking about the process of change of those special entities that we call clocks, which we use as a reference to measure the processes of change of all the other evolving entities.

transformations is even more unexpected and, in a sense, remarkable: clocks, when they move, run more slowly in comparison to clocks that are not moving. In other words, if it is true that the Galilean relativity has indicated (although this has never been really taken sufficiently seriously) that we don't live in a single three-dimensional container, as each entity "constructs" its own space, Einsteinian relativity adds to the picture the fact that these spatial theaters are not just spatial, but spatiotemporal.

11 A 4-dimensional representation

That each classical (in the sense of non-quantum) entity can be viewed as immersed in a four-dimensional spatiotemporal representation is in fact easy to demonstrate. For this, let us consider the following *gedankenexperiment* [see (Aerts, 1999), for more details]. Imagine being in Geneva, Switzerland, and that it is May 18, 2017, 3 pm. Let us call this the present moment t_0. When in Geneva, at time t_0, since you are having a direct experience with the city, you can safely affirm that Geneva is real for you, i.e., that Geneva is an existing part of your present material reality. But what about the reality of the city of Miami, at the same time t_0? Since you are not having an experience with the city of Miami, can you nevertheless affirm that it is also part of your present reality, at time t_0? The answer is affirmative, and the reason for this is that, following EPR's reality criterion, we know that *reality is a construction about the possible*: if, in your past, you would have decided to travel to Miami, then *with certainty* you would have had a direct experience with Miami at the present time t_0, and based on the certainty of such a prediction you are allowed to affirm that also Miami is an existing part of your material reality, at time t_0.

But consider now Geneva not at time t_0, but at subsequent time $t_1 > t_0$, where t_1 is, say, May 19, 2017, 3 pm,[8] that is, twenty-four hours in your future with respect to the present time t_0. Is Geneva

[8] This is the day when the *2nd International Congress on Consciousness* started in Miami, where the content of this article was presented by one of the authors.

at time t_1 part of your present material reality? The answer we would give to this question, based on our parochial (Newtonian) conception of space and time, is that this cannot be the case, as Geneva at time t_1 being in your future, it cannot be already real, i.e., part of your present material reality. But this would be a wrong conclusion considering what we know about relativistic effects and more particularly about the effect of *time dilation* (the slowdown of the ticking rate of clocks when they are moving, as compared to clocks at rest).

More precisely, if in the past, say on May 17, 2017, 3 pm, you would have decided to use your space ship to travel at a speed $v = \sqrt{3/4}\,c \approx 0.866\,c$, with c the speed of light,[9] then by performing a suitable round-trip journey to any spatial direction, and because of the effect of time dilation, you could have been back in Geneva exactly when your smartphone would indicate May 18, 2017, 3 pm, whereas the smartphones of all other Geneva's inhabitants would indicate May 19, 2017, 3 pm. Thus, if you take seriously the EPR's reality criterion, you are forced to conclude that Geneva in twenty-four hours is also part of your present material reality.

Let us explain to the reader who is less versed in relativity theory how this time dilation effect can be calculated. We have two different versions of you, one remaining always at rest with respect to the Geneva's referential frame,[10] let us call it the *A*-version of yourself (or simply the *A*-entity), and the other one who, at some moment in the past, decides to use a space ship, let us call it the *B*-version of yourself (or simply the *B*-entity). If T_B is the time-period of the clock used by *B* when traveling with the space ship, as measured by *A*, then when comparing it to the time-period T_A of an identical clock remaining in Geneva, *A* will observe an *effect of time-dilation*. More precisely, if v is the speed with which *B* moves away from *A*, or approaches *A*, then, defining the so-called *Lorentz gamma factor* $\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$ (a number strictly greater than

[9] Assuming that such spaceship, which is not forbidden by the laws of physics, would be already available.

[10] Of course, Geneva, being part of planet Earth, cannot really be associated to an inertial frame, but for simplicity we will neglect the planet's non-uniform motion.

1, if $v > 0$, whose value in our example is precisely $\gamma = 2$), we have the relativistic time-dilation formula: $T_B = \gamma T_A$, and more specifically in our example $T_B = 2T_A$, i.e., the clock on board the ship appears to *A* to run twice as slow than the clock that remained on Earth.

Imagine now that the *B*-entity performs a return trip, always going at the same constant speed v (of course, there will be accelerations at the departure, turnaround and arrival, but let us neglect them to simplify the discussion). If we assume that the *A*-entity measures n_A cycles of his clock for the entire duration of the *B*-entity trip, then we can ask what the duration of the trip as measured by the B-entity is. In other words: how many cycles n_B does the clock of the *B*-entity perform during its trip? For this, we have to solve the equation $n_B T_B = n_A T_A$, and because of the above time-dilation formula, we can write: $n_B \gamma T_A = n_A T_A$, from which it follows that: $n_B = \gamma^{-1} n_A$, so that, for our choice of traveling speed, we have $n_B = n_A/2$. We thus find that the traveling *B*-entity uses half the time cycles of the non-traveling one.

To calculate the time t in the past with respect to the present time t_0 (corresponding to May 18, 2017, 3 pm) at which the *B*-version of you would have needed to start its trip at the speed of $v \approx 0.866\, c$, to be back in Geneva at time t_1 (corresponding to May 19, 2017, 3 pm), with the traveler's clock indicating May 18, 2017, 3 pm, we have to reason as follows: let $n_A = (t_1 - t)/T_A$ be the number of cycles performed by the *A*-clock while the *B*-entity travels, and let $n'_A = (t_1 - t_0)/T_A$ be the number of cycles corresponding to a 24-hour period. We want the number n_B of cycles performed by the *B*-clock during his trip to be exactly $n_B = n_A - n'_A$ (i.e., we want the *B*-clock to use 24 hours less than the *A*-clock) and since we have $n_B = \gamma^{-1} n_A$, we can easily solve the previous equation for n_A. This gives: $n_A = n'_A/(1 - \gamma^{-1}) = 2n'_A$. In other words, the *B*-entity has to start her travel two days before May 19, 2017, 3 pm, that is, at a time t corresponding to May 17, 2017, 3 pm (see Figure 3).

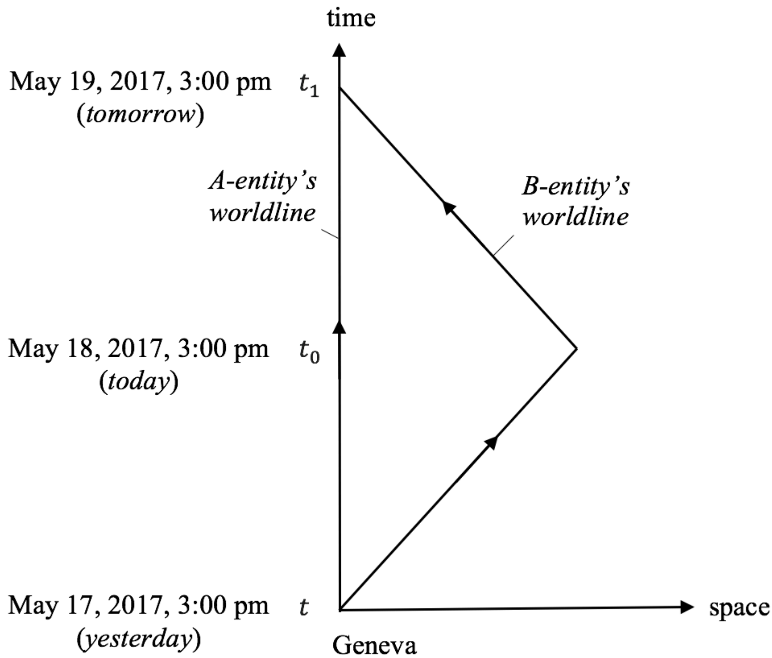

Figure 3 The two worldlines of the *A*-entity and *B*-entity, in the spacetime associated with the former. The *A*-entity remains at rest, thus only moves along her time axis, whereas the *B*-entity goes for a round-trip journey, at constant speed $\boldsymbol{v}$, which allows her to meet again with the *A*-entity, in her future.

Therefore, as we said already, considering that our reality is defined in a *counterfactual way* (by means of the EPR criterion), we are forced to accept that our present also contains part of our future, as a consequence of the existence of the *generalized spatiotemporal parallax effects* that the Einsteinian relativity has unveiled to us. Although these are just perspective effects, describing *appearances*, they are not any less real, as many experiments have clearly demonstrated. Just to give an example, a *muon* (an elementary particle similar to the electron) has a *mean lifetime* which is of the order of a *microsecond* (10^{-6} seconds), if measured in a referential frame in which it is at rest. Muons that are produced by cosmic rays in the upper atmosphere, however, can travel at speeds close to the speed of light, with respect to Earth's reference frame. If one calculates the average distance they should be able to cover, if we don't take into account the time-dilation relativistic effect, it should only be of a few hundred meters. However, because time-dilation effects are 'real appearances', they actually have (from our earthly perspective) a much longer mean lifetime, and are able to travel distances of several kilometers, in perfect accordance with the relativistic formulae.[11]

Coming back to our general discussion, if it is true that Galilean relativity has already told us that the physical entities are not immersed in a substantive objective space, but that each one "constructs" (or "sees") a personal different 3-dimensional relational space, Einsteinian relativity pushes this even further, telling us that entities are not immersed in a substantive objective space and time, but that each one "constructs" a personal and different 4-dimensional spacetime. This means that time is now viewed in a way that becomes much more similar (although not equivalent) to space, i.e., as a way for each entity to represent how the other physical entities evolve and the possible encounters they can have with them.

What is relevant for the thesis we are presenting in this article is that if it is true that relativity points to the existence of different spatiotemporal representations associated with the viewpoints of the different evolving physical entities, this also automatically points to the existence of an underlying non-spatial and non-temporal reality. So, similarly to quantum mechanics, relativity becomes truly understandable only if one is bold enough to introduce the hypothesis that physical entities are essentially non-spatial and non-temporal. And as we are now going to explain, the view that physical entities would have primarily a conceptual nature does not only offer an explanation for the strangeness of the quantum effects, but also for the relativistic ones, which by the way are usually erroneously considered to be less surprising than the former. But before this, and for sake of clarity, a remark is in order.

When we observed that 'Geneva tomorrow' is also part of your present reality, this shouldn't be understood in the sense that the future would already be given. In every moment, our reality is shaped by continuous acts of creation. For instance, this article,

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[11] This is our perspective from Earth's reference frame. From that of a reference frame associated with the muon, what will be observed instead is a (Lorentz) *length contraction*, which can equivalently explain the unusual survival over distances of the relativistic muon.

before it was written by its authors, it was not part of their present reality, despite the fact that it is now a fully created entity which is available to be part of the readers' experiences. In other words, what we are describing is not an unchanging *block universe*, where everything would be given once for all, because time would possess an ontology similar to space. In fact, our view is telling us precisely the opposite: that spacetime is just how the physical entities can represent, each one in a different way, a portion of the processes of creation and discovery in which they participate. Although in a given reference frame we can always attach a time and space coordinate to a given event, this doesn't mean that the processes of change that have given rise to it are also in space and time. In our view, they typically originate from an underlying reality that is genuinely non-spatiotemporal. But non-spatiotemporal does not mean that it would not be a *process reality*, where entities would not be able to change state, interact and evolve, in both deterministic and indeterministic ways. It is just that this non-spatiotemporal reality is a more abstract realm, which remains hidden from our parochial spatiotemporal perspective.

12 Explaining time dilation

Let us come back to our example of the two versions of you, one remaining at rest with respect to Geneva and the other one performing a return trip. If these two versions are considered to be two twins, we have the well-known situation of *Langevin's twin-paradox*, where the twin who makes the journey, when he returns home, discovers that his brother has aged more than him. The reason why it is called a paradox is not the observation of this age difference, but the fact that one could argue that if we consider the viewpoint of the reference frame associated with the space ship, then it is the twin remaining on Earth that appears to perform the return trip. However, this apparent symmetry between the two descriptions can be easily broken by observing that the two frames of reference are non-equivalent, as the frame associated with the twin traveling with the spaceship is a non-inertial one. Thus, the symmetry is broken by observing that the traveling twin experiences accelerations that are not experienced by the non-traveling one.

If the presence of accelerations allows one to eliminate the paradox, one should not conclude for this that the observed time-dilation effect (or length contraction effect, from the viewpoint of the traveling twin) would be caused by these accelerations. It is in fact not difficult to convince oneself that it is really the geometric structure of the *worldlines*[12] associated with the two brothers that is responsible for the time dilation effect, as the latter is truly defined by the (Lorentz-invariant) length (corresponding to the so-called proper time interval) of their worldlines (Aerts, 2018).

But how can we understand this very strange effect of time dilation? For this, imagine that the two twins, *A* and *B*, are not just 'bodies moving in space', but, at a much more fundamental level, 'mind-like entities having some meaning driven interactions'. Assume that they both start reflecting on a problem, at time t, on a given hypothesis about which they both agree. In other words, in the conceptual space that they both inhabit, they have a first encounter at the "place" of this commonly shared premise. Imagine then that entity *A*, after n_A *conceptual steps*, reaches a given conclusion, and that to keep track of her cognitive path, *A* decides to introduce an axis to parameterize each one of her n_A conceptual steps. By ascribing a unit to this axis, corresponding to the length L_A of a single conceptual step, then assuming that the duration of such step is T_A, and that the speed with which it is accomplished is *c*, we can simply write: $L_A = cT_A$. Thus, going through her reasoning, from the hypothesis to the conclusion, *A* performs n_A conceptual steps, each one of length L_A, thus moving on her 'order parameter axis' from point $L = ct$ to point $L_1 = ct_1 = L + n_A L_A = c(t + n_A T_A)$.

Consider now entity *B*. Differing from entity *A*, we assume that her reasoning allows her to reach the same conclusion (starting from the same hypothesis), but in a lesser number $n_B < n_A$ of conceptual steps, and let us consider for simplicity that, as in our previous example, $n_B = n_A/2$. Imagine that entity *A* is also willing to keep track of the cognitive path of entity *B*, consistently with the

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[12] The worldline of an entity is the path traced in the 4-dimensional spacetime describing the history of its location in space at each instant in time.

fact that they are at the same "place" when they share the starting hypothesis, and that they can meet again at a common "place" when they reach the same conclusion. We assume that *A* and *B* are equivalent entities, in the sense that when they take a cognitive step, they always do so at the same speed *c*.

Now, since *B* is able to reach the same conclusion in half the number of steps of *A*, the latter cannot represent her path on the same axis, as units were precisely chosen on the latter in a way that one needs twice the number of steps to reach the final conclusion. So, *A* has to find a different way to represent the cognitive process of *B*, by introducing an additional axis, to describe *B* as moving on a round-trip path which will be now contained in a higher dimensional space, generated by the first parametric axis. Let us call it the 'time axis of *A*', and this second parametric axis, let us call the 'space axis of *A*'.

So, entity *B* is now described as following a conceptual path that moves away from the initial "hypothesis point," on the time axis, and then comes back to reach the "conclusion point," always located on the time axis, by doing exactly $n_A/2$ cognitive steps. However, if we consider this construction from a purely Euclidean perspective, we immediately see that there is a problem. Indeed, if we calculate the length of the *B*-path using the Pythagorean theorem, we will necessarily find a longer path than that walked by *A* (see Figure 4).

But we know that *B* follows a shorter conceptual path, as is clear that she only uses half of the conceptual steps used by *A*. Accordingly, when measuring its conceptual length, this should be shorter and not longer than that of the path followed by *A*. For *A* to fix this problem, there is only one way to go: it has to consider a *pseudo-Euclidean* space, instead of a Euclidean one, and more precisely that specific pseudo-Euclidean space known as the *Minkowski space* (or spacetime). In the latter, distances are not calculated using the usual *Pythagorean theorem*, but using a *pseudo-Pythagorean theorem* that attaches a negative sign to the squares of the components associated with the space axis, and a positive one to the square of the components associated with the time axis. In this way, the length of the hypotenuse of a right triangle, whose catheti are associated with the time and space axes, respectively, will generally be less than the length of the time-cathetus.

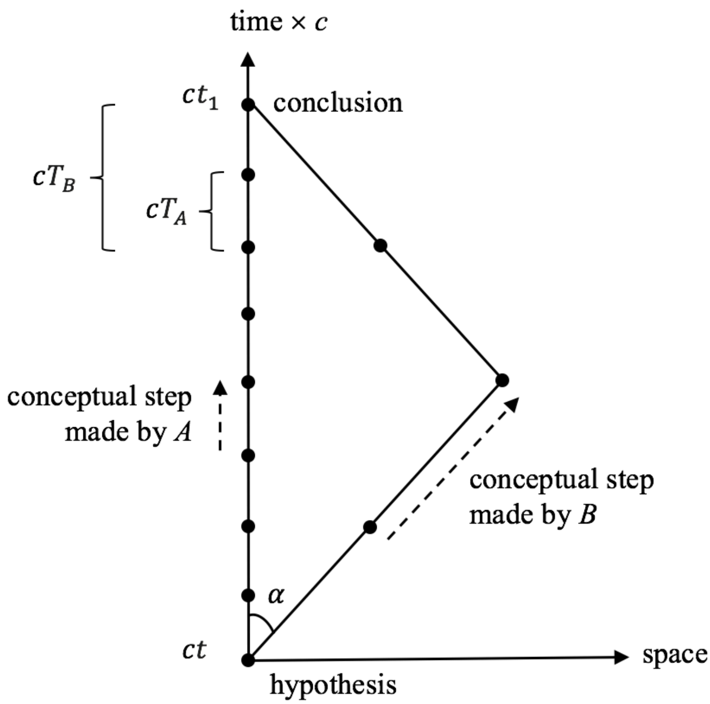

Figure 4 The coordination of the conceptual paths followed by the two entities *A* and *B*, in the spacetime constructed by the latter. When measured along the *A*-time axis (here multiplied by the light speed *c*) the length of the conceptual steps acted by *B* appear to be longer than those acted by *A*. However, when measured along the direction of its own movement in the *A*-spacetime, using the Minkowski instead of the Euclidean metric, one finds that the two entities' conceptual steps are exactly of the same length, in accordance with the fact that they would both move at the same speed *c* in the underlying conceptual reality.

Thanks to this pseudo-Pythagorean theorem, it becomes possible for the length L_B of a single conceptual step of *B* (see Figure 4) to be exactly equal to the length L_A of a single conceptual step of *A*, i.e., to have the equality $L_B = L_A$. This is a must because, as we said already, we are assuming that the two entities *A* and *B* move at the same speed *c* in their common conceptual reality, and that the duration of a single conceptual step is an *invariant*, i.e., is the same for all entities.

According to the pseudo-Pythagorean theorem, if L is the space component of the length of a conceptual step of *B*, relative to the space-axis of *A*, we have: $L_B^2 = (cT_B)^2 - L^2$. Considering that $L_B = L_A$ and $T_B = \gamma T_A$, a simple calculation shows that: $L = vT_B$. In other words, entity *A*, by adopting a pseudo-Euclidean (Minkowski) metric, is able to construct a spacetime theater in which she can keep track, in a consistent way, not only of its cognitive processes, but also of those of other entities.[13] For this, all it has to do is attach an appropriate spatial velocity v to them.

So, from the viewpoint of the underlying conceptual realm, the reason for the time-dilation effects becomes now very clear: since *A* has to parametrize the "cognitive movement" of *B*, and to do so the latter must have an angle $\pm\alpha$ with respect to the direction of the cognitive movement of *A*, with $\alpha = \tan^{-1}\frac{v}{c}$, inevitably there will be spatio-temporal parallax effects: *A*, form her viewpoint, will see *B* as if she was producing cognitive steps (cycles) having an increased duration $T_B = \gamma T_A$, that is, will see *B* as if she was reasoning more slowly than her, but since she would also reason more efficaciously (by using a lesser number of cognitive steps), they can nevertheless meet again at the conclusion point. This, however, is just how things *appears to be* at the level of the spacetime construction that is operated by *A*. At the more objective level of the non-spatiotemporal conceptual space, in which both *A* and *B* are immersed, they would both move at exactly the same speed *c*:[14] the intrinsic speed at which they both perform their conceptual steps.

13 Quantum and relativity

Our description of time-dilation effects would require more explanation to make it fully intelligible, but this we cannot do in the limited space of this article. So, we refer the motivated reader to (Aerts,

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13 When considering only two entities, one does not need more than a single space dimension to represent the second entity in the spacetime of the first one. However, additional space axes become necessary if one consider additional entities; for more details, see (Aerts, 2018).

14 In the relativistic formalism, the clue that all entities actually move at the intrinsic velocity *c* can be found in the mathematical fact that the length of the *four-velocity vectors* is always precisely equal to *c*.

2018), where more details about this construction can be found, which surely begs to be further explored and investigated to be fully understood.

Our main point here was to emphasize that relativity theory, similarly to quantum mechanics, points to the existence of a non-spatio-temporal realm, probably of a conceptual-like nature. The non-temporal aspect of this deeper conceptual reality should not be understood, however, in the sense that nothing would ever change in it. On the contrary, every physical entity would constantly surf over it, at the light speed c, continually producing new conceptual steps. So, movement would be incessant, and in a sense absolute, at this more fundamental level. However, time and space would be absent there, because they would only emerge when an entity coordinates aspects of her and others' movements by introducing a specific coordinate system, first by considering a time axis, to give an order to its own conceptual steps, then by introducing additional spatial axes, to put these personal steps in relation to those accomplished by the other physical entities. When this is done, as we have tried to outline in the above description of the traveling twin, one observes that movements in the conceptual reality cannot be mapped into movements happening only in space, but also in time (i.e., also along the time axis), in a way that can certainly affect how an entity can reach out into the future, explaining why not only 'Geneva now', but also 'Geneva tomorrow', can be part of one's present reality.

One may ask: If the Minkowskian construction is the one used by all physical entities (not just observers) to create a common space of encounters, then why does it remain so counterintuitive to us humans? This is because we have evolved on this planet mostly being surrounded by entities moving extremely slowly in space with respect to one another. In other words, our ordinary reality is formed by spatial entities that are almost at rest with respect to one another, so that the relativistic spatiotemporal parallax effects have remained mostly unnoticed, and therefore have not been integrated in our mental representation of the world. The same is true for the quantum effects, which mankind only discovered very recently, thanks to our accurate laboratory experiments, particularly at the microscopic level. What is fascinating is that both relativity and quantum mechanics point to the existence of a non-spatial and non-temporal (more fundamental) layer, which is the ambit that probably needs to be considered when attempting to construct a fully consistent theory of *quantum gravity*.

Now, when considering quantum entities, instead of classical ones, the duality between 'time' and 'space' needs to be replaced by a duality between 'time' and the 'set of outcome states' of a measurement. This will introduce the additional ingredient of indeterminism, as the actualization of an outcome generally involves a symmetry breaking process (Aerts & Sassoli de Bianchi, 2016b).

We have already mentioned in this article the *extended Bloch representation* (Aerts & Sassoli de Bianchi, 2014, Sassoli de Bianchi, 2015), which can be used to construct a *quantum theater* in which the measurement processes of a quantum entity, and the associated sets of outcome states, can be fully represented. In the simple example of a spin one-half entity, only admitting two-outcome measurements, the quantum theater requires only three dimensions, but for more complex entities, admitting sets of *n* possible outcomes, the number of required dimensions appears to be equal to $n^2 - 1$. Technically speaking, this corresponds to the *number of generators of the so-called SU(n) group of transformations*, which can be roughly interpreted as a group of "generalized rotations." This means that to enter the quantum (Blochean) theater, where measurements can be represented, one has to somehow "rotate away" the inherent complexity of a quantum entity, by means of these generators, in a way which will depend on its specific state.

The spin observables of a spin one-half entity, the so-called *Pauli matrices*, are in fact a particular example of these generators, for the $n = 2$ case. Thus, one can also speculate that spin is precisely an example of that very special "twist" one needs to add to a physical entity to allow it to enter a spacetime theater. As we have tried to explain, spacetime would be a creation produced by the "surfing" of the entities over a more fundamental conceptual-like reality. The latter, however, would be a much more complex and higher dimensional reality than the one representable in a global spacetime theater. This greater complexity and higher dimensionality manifests first of all in the fact that each entity must bring with it its personal spacetime construction. This however would still not be sufficient for representing also the evolution of the other entities. For this, their complexity has to be in part singled out ("rotated away"), and spin might very well be the manifestation of this "subtraction operation," allowing the quantum entities to manifest in spacetime theaters.

Let us offer a simple analogy from human cognition, to better explain what we mean by this. We can compare a spacetime theater with a well-defined 'space of discourse', like for example a given political agenda. In order to be able to fit individual words or sentences into this space of discourse, they will have to carry some specific "twists," which will make them appropriate to be part of it. These "twists" would play the same role in allowing certain terms to enter a given human space of discourse as the role played by the "spinorial twist" in allowing quantum entities to become available to enter a given spacetime theater.

14 Perspectives

Approaching the end of this essay, let us offer some more speculative perspectives, which however can be seen to be the logical consequence of what we have so far described. In that respect, let us recall that the content of the present article has been presented at the *2nd International Congress on Consciousness*, which took place in Miami, Florida (USA), from May 19 to 21, 2017. The idea behind this congress is that of promoting an open exchange and debate on research centered upon the consciousness, with particular emphasis on the importance of a *multidimensional paradigm* to explain the numerous phenomena related to the manifestation of the consciousness, like *extra-sensory perceptions* (ESP), *psychokinetic phenomena* (PK) and *extracorporeal phenomena* (OBE and NDE).

In (Sassoli de Bianchi, 2015), one of us proposed to consider, in addition to the well-known easy and hard problems of consciousness, as defined by Chalmers (1995), a *serious problem of consciousness*, which is about the identification of models and mechanisms able to explain the aforementioned *parapsychic phenomena*. The adjective "serious" is to be understood in the double sense of indicating that this is undoubtedly a difficult problem, but also a problem that requires taking seriously not only the ordinary manifestations of the consciousness, but also the (apparently) extra-ordinary ones.

There are many reasons why the 'serious problem of consciousness' is currently not openly addressed in academia, but it is not our intention to enter into this delicate and highly polarized debate. Let us mention however what is probably the most commonly evoked one, which is summed up in *Carl Sagan*'s famous quote, saying that: "extraordinary claims require extraordinary evidence." One should however ask: What determines the extraordinariness of a claim? Undoubtedly, this depends on how easily it can be integrated in the *dominant worldview*, that is, in the commonly adopted conceptual map that is used to construct a global image of the world and of the aspects that we think we can experiment with.

If we adopt the (false) *parochial worldview* of a reality fully contained in a three-dimensional substantive space, then surely claims about the reality of, say, *telepathic and precognitive phenomena*, will be evaluated being as extraordinary as claims about the reality of flying unicorns. However, if we adopt a less parochial view, like the one of multiplex realism that we have presented and motivated in this article, then phenomena like telepathy and precognition can be considered to be rather ordinary, and in fact the extraordinary claims requiring extraordinary evidence would become those affirming that parapsychic phenomena could not be real.

In a sense, we can say that if parapsychic phenomena are generally evaluated as unreasonable phenomena by the majority of modern scientists, it is because, notwithstanding the quantum and relativistic revolutions, they are still maintaining an antiquated 'Newtonian-like worldview'. However, if we take quantum physics seriously, then we must accept that our physical reality is fundamentally non-spatial, and if we take relativity seriously, we also have to accept that reality is fundamentally non-temporal; and if we reflect attentively about the behavior of quantum and relativistic entities, we must surrender to the fact that it is much more similar to that of concepts than that of objects.

But then, if it is true that reality, at a deeper level, is non-spatiotemporal, each time the state of a physical entity collapses into a localized spatiotemporal state, the process will necessarily produce correlations both over space and over time, as is clear that a non-spatiotemporal state (a non-local state both in space and time) will generally describe a superposition over different spacetime regions.

Consider for instance precognition, i.e., the possibility of "seeing" future events. There is of course a "down-to-earth" way to know about future events, which is that of using knowledge about the initial condition of a system and the laws governing its evolution. If these laws are deterministic, future events can be seen, in principle, with arbitrary accuracy, whereas if they are indeterministic, as in quantum measurements, then the future will only be available (predictable) in probabilistic terms. As an example, consider a cat sitting on your sofa. If you caress it, you could predict with great confidence that soon it would start purring. This is a prediction based on the description of a process happening in the ordinary spatial theater, i.e., at the level of our "ordinary macroscopic material reality." However, we should not forget that without a human mind producing an abstraction it would be impossible to make a prediction in the first place; so, even for such a "down-to-earth" way of understanding precognition, when everything is already collapsed into the ordinary material space, it would be wrong to consider that a more abstract cognitive-like realm would not also be involved in the process.

We can call this more abstract reality, needed to produce ordinary predictions, the "down-to-earth-mind." The time-like causal connection that the "down-to-earth-mind" is able to reveal is not however one that can produce genuine precognitive phenomena, as usually understood. Indeed, our "down-to-earth-mind" (that part of our human mind that we use in our everyday intraphysical life, to move around, take ordinary decision, etc.) can only penetrate very coarsely and in a very limited way into the more abstract 'meaning realm' of our physical reality. However, it is not unreasonable to consider that another part of our mind – let us call it the "up-in-the-sky-mind," mostly manifesting at the subconscious level – could access this more abstract domain of potentiality, where superposition states have not yet collapsed into more specific spatio-temporal instances.

The non-ordinary "up-in-the-sky-mind" could for instance be associated with those quantum-like properties of our brain that cannot be accounted for by the usual classical-computer brain models. It could also be related, more generally, to our bodies, considering that physical entities only have the appearance of objects moving in space and time, whereas at a more fundamental level they would manifest more like meaning-entities exploring a vaster conceptual reality. And of course, the non-ordinary "up-in-the-sky-mind" could also be related to more subtle/abstract structures yet to be put in evidence in experimental terms, which also could penetrate more deeply into the multidimensional fabric of the non-spatiotemporal realm.

So, in principle a physical entity can have access not only to superposition states over different regions of space, at a given time (creating correlations between causally separated events, thus introducing an element of *synchronicity* in our spatial reality), but also to superposition states over different times. The latter can produce the usual correlations in time, an expression of the fact that what happens in the present will generally affect the future (causality), but this certainly does not exhaust all possibilities. The spatiotemporal reality can only realize some of the possibilities in terms of 'connections based on meaning', and also the quantum correlations could be considered to be just a subclass of all possible correlations, resulting from all possible 'connections through meaning' characterizing our huge multidimensional reality.

Take the collection of pages of the world-wide-web as a metaphor for the different locations in space, each webpage representing a different spatial location. Beyond this structure, there is another web, much more abstract and much more fundamental, which is that of the *meaning content* associated with the ordinary world-wide-web. When you move from one page to another, by clicking on the different hyperlinks, you move in an already "collapsed reality," and can just explore a subclass of all possible *meaning connections*: those available as actually clickable hyperlinks. But these clickable hyperlinks are only a pale reflection of the entire meaning content of the web. In other words, when you are on a given webpage, its hyperlinks will connect to webpages that are still very close in meaning to its content. These hyperlinks constitute the possible futures of the webpage in question, explorable by an entity surfing the web in a causal-like way. What we have called the "down-to-earth mind" would only operate at the level of the webpages, and thus will only be able to predict the future based on the exploration of the existing "causal lines" associated with the processes of clicking on the available hyperlinks.

On the other hand, if we assume that there is a deeper level of our mind, having an access to the level of the meaning connections that have not, or not yet, formed a concrete hyperlink, then, by contemplating these hidden connections, it would be possible to make predictions based on this wider "bird perspective," which also includes superposition/entangled states between present and future happenings. And this may explain why human minds, in certain conditions, can actually experience precognitions. Therefore, these would not be so different after all from predictions based on causality: they would simply originate from the possibility of accessing a more abstract web of connections, beyond those that are describable in spatiotemporal/material terms.

Physicists, so far, have mostly studied correlations originating from states that are non-local in space (non-spatial), and are only starting to study correlations originating from states that are non-local in time (non-temporal). Quoting from (Musser, 2016): "Normally physicists think of […] correlations as spanning space, linking far-flung locations in a phenomenon that Albert Einstein famously described as 'spooky action at a distance'. But a growing body of research is investigating how these correlations can span time as well. What happens now can be correlated with what happens later, in ways that elude a simple mechanistic explanation. In effect, you can have spooky action at a delay."

Parapsychic phenomena like the precognitive ones, could just be an example of phenomena produced by these 'entangled states in time', which from our parochial spatiotemporal perspective is perceived as "spooky actions at a delay," i.e., correlations that would be created out of meaning-connections that we could usually access only in a subconscious way, and which should be considered to be as real as the more ordinary connections based on cause-effect and action-reaction relationships.

To conclude, we have to observe that the quantum and relativistic revolutions have not yet been fully integrated in our modern worldvision, still predominantly based on spatiotemporal and mechanistic models that are certainly inadequate to account for all known phenomena. An extended worldview, which in part we have tried to delineate in this article, is however gradually gaining ground, although it is still perceived to be highly non-intuitive by the majority of physicists. It is this extended worldvision that we need to adopt if we want to have a chance at understanding the complexity and richness of our world, both at the physical and psychical (consciential) level. Then, many phenomena that currently appear to us as extraordinary, and therefore difficult to believe, may suddenly look very "down-to-earth." This would be so because we would have brought earth back to its original place: in the depths of the sky.

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